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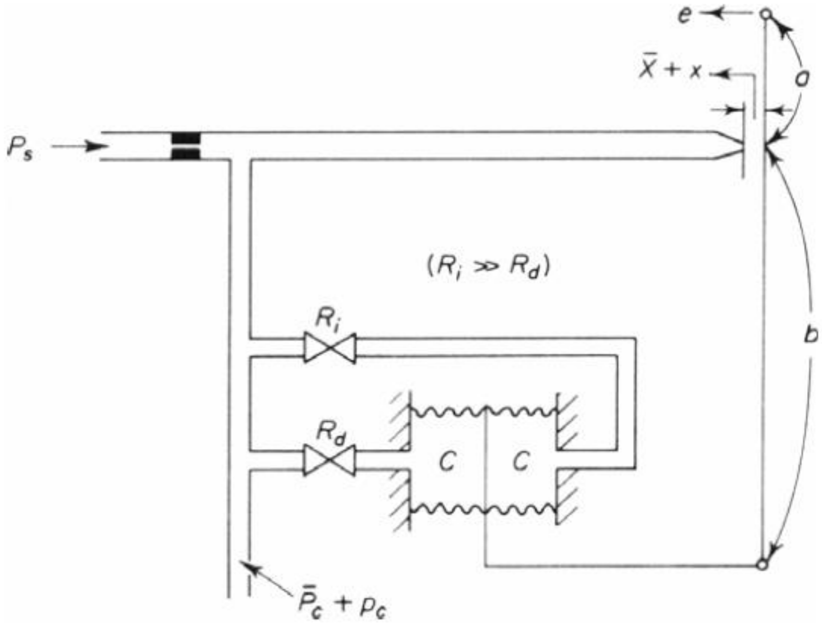

HOMEWORK #8

PNEUMATIC PID CONTROLLER

ME 411: System Modeling Analysis and Control

Fall 2010

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November 24, 2010

a) Governing equation and transfer function:

I. The pneumatic nozzle-flapper amplifier:

A schematic diagram of a pneumatic nozzle-flapper is shown in figure1.

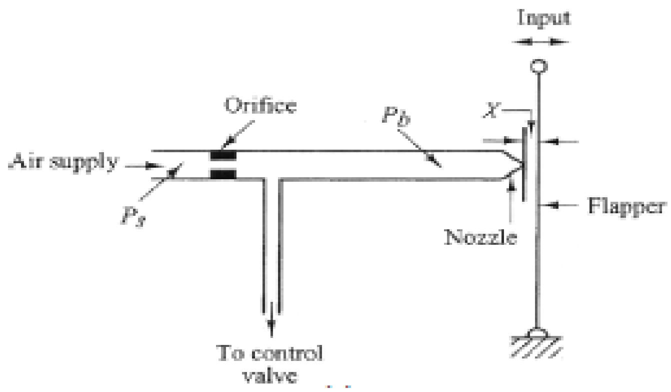


Figure 1: Pneumatic Nozzle-Flapper Amplifier

In operating this system, the flapper is positioned against the nozzle opening. The nozzle back pressure P_b is controlled by the nozzle-flapper distance X. As the flapper approaches the nozzle, the opposition to the flow of air through the nozzle increases, with the result that the nozzle pressure P_b increases.

Assuming that the relationship between the variation in the nozzle back pressure $p_b(t)$ and the variation in the nozzle-flapper distance $x(t)$ is linear, we have

$$p_b(t) = K.x(t)$$

where K is positive constant.

Since the smooth pipes with back and control pressures are connected directly, we have:

$$p_b(t) = p_c(t) = K.x(t)$$

By the Laplace transformation

$$P_b(s) = P_c(s) = K.X(s)$$

Again, if $p_i(t)$ and $p_o(t)$ are considered the input and output, respectively, then for the small values of $p_i(t)$ and $p_o(t)$, the resistance R given by , where

$$R = \frac{p_i(t) - p_o(t)}{q} \rightarrow q = \frac{p_i(t) - p_o(t)}{R}$$

And the capacitance C is given by

$$C = \frac{dm}{dq}$$

Since, the pressure change dp_o times the capacitance C is also equal to the gas added to the vessel during dt seconds,

$$Cdp_o = qdt$$

$$C\frac{dp_o}{dt} = q$$

$$C\frac{dp_o}{dt} = \frac{p_i(t) - p_o(t)}{R}$$

$$RC\frac{dp_o}{dt} = p_i(t) - p_o(t)$$

By analogy, for pneumatic resistance $R_i\ and\ R_d\ of\ resistriction\ valves$

$$R_iC\frac{dp_i}{dt} = p_c(t) - p_i(t)$$

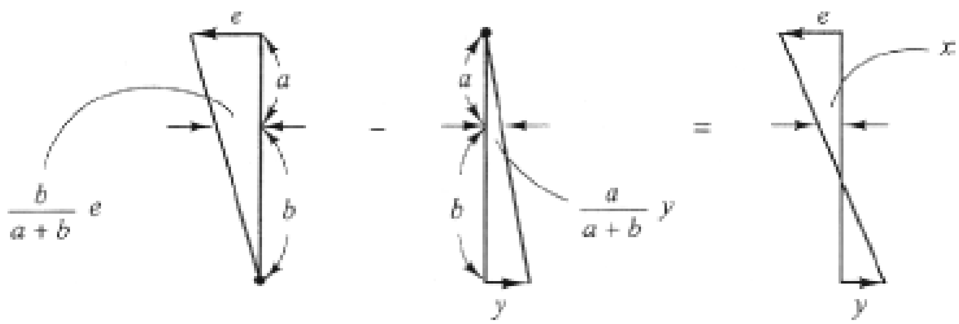
$$R_dC\frac{dp_d}{dt} = p_c(t) - p_d(t)$$

Therefore the transfer function of the subsystem is

$$\frac{P_i(s)}{P_c(s)} = \frac{1}{R_iCs + 1}$$

$$\frac{P_d(s)}{P_c(s)} = \frac{1}{R_dCs + 1}$$

As show in the figure below, we can consider such movement separately and add up the results of the two movements into one displacement, thus fo the flapper movement, we have



$$x(t) = \frac{b}{a+b}e(t) - \frac{a}{a+b}y(t)$$

$$y(t) = \frac{b}{a}e(t) - \frac{a+b}{a}x(t)$$

Assuming that the flapper acts like a spring and the following equation holds true,

$$A_b * (p_d(t) - p_i(t)) = by(t)$$

Where A is the effective area of bellows and K_b is the equivalent spring constant.

$$A_b * (p_d(t) - p_i(t)) = K_b \left(\frac{b}{a} e(t) - \frac{a+b}{a} x(t) \right)$$

By Laplace transformation

$$P_d(s) - P_i(s) = \frac{K_b}{A_b} \left(\frac{b}{a} E(s) - \frac{a+b}{a} X(s) \right)$$

Dividing both side by $P_c(s)$

$$\frac{1}{R_d Cs + 1} - \frac{1}{R_i Cs + 1} = \frac{K_b b}{A_b a} \frac{E(s)}{P_c(s)} - \frac{K_b}{A_b} \frac{a+b}{a} \frac{X(s)}{P_c(s)}$$

$$\frac{K_b b}{Aa} \frac{E(s)}{P_c(s)} = \left(\frac{1}{R_d Cs + 1} - \frac{1}{R_i Cs + 1} + \frac{K_b}{A_b} \frac{a+b}{a} \frac{X(s)}{P_c(s)} \right)$$

$$\frac{P_c(s)}{E(s)} = \frac{1}{\frac{A_b a}{K_b b} \left(\frac{1}{R_d Cs+1} - \frac{1}{R_i Cs+1} + \frac{K_b}{A_b} \frac{a+b}{a} \frac{1}{K} \right)}$$

$$\frac{P_c(s)}{E(s)} = \frac{1}{\left(\frac{A_b a}{K_b b} \frac{1}{R_d Cs+1} - \frac{A_b a}{K_b b} \frac{1}{R_i Cs+1} + \frac{a+b}{b} \frac{1}{K} \right)}$$

$$\frac{P_c(s)}{E(s)} = \frac{1}{\frac{a+b}{bK} \left(\frac{K}{a+b} \frac{A_b a}{K_b} \frac{1}{R_d Cs+1} - \frac{K}{a+b} \frac{A_b a}{K_b} \frac{1}{R_i Cs+1} + 1 \right)}$$

$$\frac{P_c(s)}{E(s)} = \frac{1}{\frac{a+b}{bK} \left(\frac{K}{a+b} \frac{A_b a}{K_b} \frac{1}{R_d Cs+1} - \frac{K}{a+b} \frac{A_b a}{K_b} \frac{1}{R_i Cs+1} + 1 \right)}$$

$$\frac{P_c(s)}{E(s)} = \frac{b}{a+b} \frac{K}{1 + K \left(\frac{a}{a+b} \frac{A_b}{K_b} \right) \left(\frac{1}{R_d Cs+1} - \frac{1}{R_i Cs+1} \right)}$$

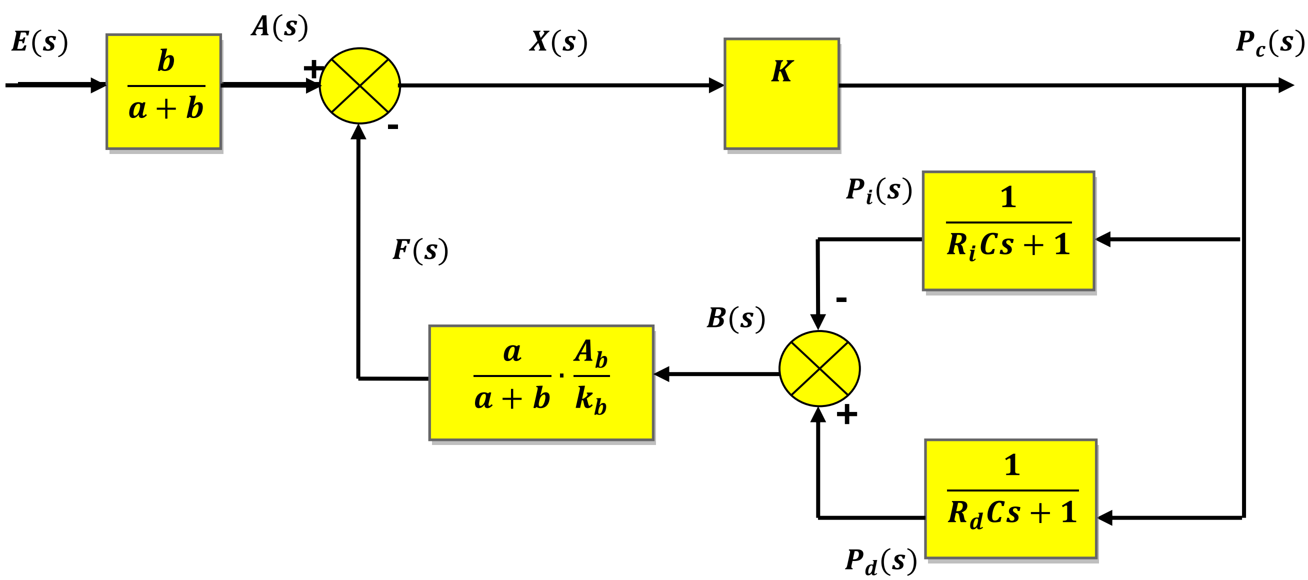
b) Block Diagram:

With the below block diagram the equation we found are:

$A(s) = \frac{b}{a+b} E(s)$ $\qquad P_i(s) = \frac{1}{R_i Cs+1} P_c(s)$ $\qquad X(s) = A(s) - F(s)$

$P_c(s) = KX(s)$ $\qquad P_d(s) = \frac{1}{R_d Cs+1} P_c(s)$ $\qquad F(s) = \left(\frac{a}{a+b} \cdot \frac{A_b}{k_b}\right) B(s)$

$B(s) = P_d(s) - P_i(s)$



$$X(s) = A(s) - F(s)$$

$$\frac{P_c(s)}{K} = \frac{b}{a+b} E(s) - \left(\frac{a}{a+b} \cdot \frac{A_b}{k_b}\right) B(s)$$

$$\frac{P_c(s)}{K} = \frac{b}{a+b} E(s) - \left(\frac{a}{a+b} \cdot \frac{A_b}{k_b}\right) \left(P_d(s) - P_i(s)\right)$$

$$\frac{P_c(s)}{K} = \frac{b}{a+b} E(s) - \left(\frac{a}{a+b} \cdot \frac{A_b}{k_b}\right) \left(\frac{1}{R_d Cs+1} P_c(s) - \frac{1}{R_i Cs+1} P_c(s)\right)$$

$$\frac{b}{a+b} \frac{E(s)}{P_c(s)} = \frac{1}{K} + \left(\frac{a}{a+b} \cdot \frac{A_b}{k_b}\right) \left(\frac{1}{R_d Cs+1} - \frac{1}{R_i Cs+1}\right)$$

$$\frac{E(s)}{P_c(s)} = \frac{(a+b)}{b} \frac{1}{K} + \frac{(a+b)}{b} \left(\frac{a}{a+b} \cdot \frac{A_b}{k_b}\right) \left(\frac{1}{R_d Cs+1} - \frac{1}{R_i Cs+1}\right)$$

$$\frac{P_c(s)}{E(s)} = \frac{1}{\frac{(a+b)}{b}\frac{1}{K} + \frac{(a+b)}{b}\left(\frac{a}{a+b}\cdot\frac{A_b}{k_b}\right)\left(\frac{1}{R_dCs+1} - \frac{1}{R_iCs+1}\right)}$$

$$\frac{P_c(s)}{E(s)} = \frac{b}{a+b}\frac{K}{K\frac{1}{K} + K\left(\frac{a}{a+b}\cdot\frac{A_b}{k_b}\right)\left(\frac{1}{R_dCs+1} - \frac{1}{R_iCs+1}\right)}$$

$$\frac{P_c(s)}{E(s)} = \frac{b}{a+b}\frac{K}{1 + K\left(\frac{a}{a+b}\cdot\frac{A_b}{k_b}\right)\left(\frac{1}{R_dCs+1} - \frac{1}{R_iCs+1}\right)}$$

c) PID Control parameters:

Assume that under normal operating conditions:

Loop gain: $|G(s).H(s)| = \left|1 + K\left(\frac{a}{a+b}\cdot\frac{A_b}{k_b}\right)\left(\frac{1}{R_dCs+1} - \frac{1}{R_iCs+1}\right)\right| \gg 1$

And $R_i \gg R_d$

Therefore mentioned transfer function can then be used to approximate a PID controller, i.e.,

$$G_c(s) = \frac{P_c(s)}{E(s)} = \begin{cases} K_p\left(1 + \frac{1}{T_I s} + T_D s\right) \\ K_p + \frac{K_I}{s} + K_D s \end{cases}$$

Then, We have

$$\frac{P_c(s)}{E(s)} = \frac{P_c(s)}{E(s)} = \frac{b}{\cancel{a+b}}\frac{\cancel{K}}{\cancel{K}\left(\frac{a}{\cancel{a+b}}\cdot\frac{A_b}{k_b}\right)\left(\frac{1}{R_dCs+1} - \frac{1}{R_iCs+1}\right)}$$

$$= \frac{b}{\left(\frac{aA_b}{k_b}\right)\left(\frac{1}{R_dCs+1} - \frac{1}{R_iCs+1}\right)}$$

$$= \frac{bk_b(R_iCs+1)(R_dCs+1)}{aA_b(R_iCs - R_dCs)}$$

$$= \frac{bk_b(R_iCs+1)(R_dCs+1)}{aA_b(R_iCs - R_dCs)}$$

$$= \frac{bk_b(R_iR_dC^2s^2 + R_iCs + R_dCs + 1)}{aA_b(R_iCs - R_dCs)}$$

$$= \frac{bk_b(R_iR_dC^2s^2 + R_iCs + 1)}{aA_b(R_iCs)}$$

$$= \frac{bk_b}{aA_b} \frac{R_i R_d C^2 s^2 + R_i Cs + 1}{R_i Cs}$$

$$= \frac{bk_b}{aA_b} \left(1 + \frac{1}{R_i Cs} + R_d Cs\right)$$

Therefore by analogy

The PID control parameters are:

$$Proportional\ gain\ (K_p) = \frac{bk_b}{aA_b}$$

$$Integral\ Time\ (T_I) = \frac{1}{R_i C}$$

$$Integral\ gain\ (K_i) = \frac{K_p}{T_I} = \frac{bk_b}{aA_b R_i C}$$

$$Derivative\ Time\ (T_D) = R_d C$$

$$Derivative\ gain\ (K_D) = K_p * T_D = \frac{bk_b}{aA_b} R_d C$$